DOCUMENT RESUME

ED 410 112 SE 060 506

AUTHOR Porzio, Donald T.
TITLE Examining Effects of Graphics Calculator Use on Students' Understanding of Numerical, Graphical, and Symbolic Representations of Calculus Concepts.
PUB DATE Mar 97
NOTE 15p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, March, 1997).
PUB TYPE Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Calculators; *Calculus; Concept Formation; *Educational Strategies; Graphing Calculators; Graphs; Higher Education; Knowledge Representation; Learning Strategies; Mathematics Curriculum; *Mathematics Instruction; Problem Solving; *Technology

ABSTRACT

This research focuses on the graphics calculator in calculus and its impact on students' abilities to work with graphical, numerical, and symbolic representations. Three courses are compared, two of which are taught with less emphasis on the use of graphing calculators. The format of the course emphasizing calculator use is more traditional than the other two and stresses symbolic and graphical representations generated via graphics calculators. The textbook used in this course presents the topics both algebraically and graphically. Course work employs three different approaches to problem solving. Findings suggest that there is a need for further research on the effects of instruction emphasizing the use of multiple representations in the presentation of concepts. Results also provide evidence that students "behave" as they are taught and that the addition of technology does not necessarily improve the learning of calculus. Contains 18 references. (DDR)

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# Examining Effects of Graphics Calculator Use on Students' Understanding of Numerical, Graphical, and Symbolic Representations of Calculus Concepts

by
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# EXAMINING EFFECTS OF GRAPHICS CALCULATOR USE ON STUDENTS' UNDERSTANDING OF NUMERICAL, GRAPHICAL, AND SYMBOLIC REPRESENTATIONS OF CALCULUS CONCEPTS

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How can graphics calculators be integrated into the mathematics curriculum so as to enhance the teaching and learning of mathematics? This question has been the focus of much research since the National Council of Teachers of Mathematics, in their *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) proposed that, in the grade 9-12 mathematics curriculum, "[S]cientific calculators with graphing capabilities will be available to all students at all times" (p. 124). At the collegiate level, research has investigated the effects of graphics calculators and computer graphing utilities in courses ranging from college algebra (e.g., Pankow, 1995; Hollar, 1997) to precalculus (e.g., Browning, 1988; Dunham, 1991; Tuska, 1993) to calculus (e.g., Aspinwall, 1995; Emese, 1993; Stiles, 1995). In the area of calculus, the research has concentrated mainly on the impact of the technology on the development of conceptual and procedural knowledge (e.g., Boers & Jones, 1993; Lauten, Graham, & Ferrini-Mundy, 1994). Less research has been done on the impact of technology use on other aspects of students' understanding of calculus, such as their understanding of different representations (numerical, graphical, or symbolic) of calculus concepts (Beckmann, 1989; Ellison, 1994; Hart, 1991). During the 1993-1994 school year, a study was undertaken to examine the effects of three different instructional approaches to calculus - one traditional, one involving the use of graphics calculators, and one involving use of Calculus & *Mathematica* (Davis, Porta, and Uhl, 1994) - on students' abilities to use and understand connections between numerical, graphical, and symbolic representations when solving calculus problems. This paper focuses on the graphics calculator calculus course and the impact of the graphics calculator use in that course on students' abilities to work with graphical, numerical, and symbolic representations.

## Theoretical Framework

A framework derived from the theories of Hiebert and Carpenter (1992) and Dubinsky (1991) was developed to help analyze differences in students' abilities to use and understand connections between numerical, graphical, and symbolic representations. Hiebert and Carpenter propose a theoretical framework for defining understanding. This framework is based on the premise that knowledge is represented internally or mentally, internal representations can be connected, and internal representations, along with any associated connections, form networks of represented knowledge. They theorize that a mathematical fact, idea, or procedure is understood if its internal representation is part of a network of represented knowledge and that the degree of understanding is determined by the number and strength of the connections to this representation in the internal network containing the representation. Under this framework, differences in students' abilities to use or recognize connections between numerical, graphical, and symbolic representations of a concept can be analyzed in terms of differences in internal networks of represented knowledge likely to be formed by students given the type of instruction they received in their calculus course.

Dubinsky (1991) extends Piaget's notion of reflective abstraction to advanced mathematical thinking to develop a theory about the construction of mathematical knowledge. Dubinsky contends that when students are solving problems, reflective abstraction occurs as they construct new knowledge associated with the problem and its solution. If a problem is solved successful, then a student will somehow assimilate the problem and solution into one or more existing schema. If the problem is not solved successful, then the student may or may not make accommodations in existing schema to deal with the unsolved problem. Dubinsky's (1991) theory was incorporated into the theoretical framework for this study because of the observed dissimilarities in the types of problems students from the different courses were asked to solve. This portion of the framework was useful for explaining differences in students' use and understanding of numerical, graphical, and symbolic representations in situations where the differences could not be explained through analysis of the internal networks of knowledge students were likely form based upon the instructional approach they experienced.

## Environment

Participants in the overall study were taken from intact classes from three calculus courses at a large midwestern university. Each course was the first in a four-quarter sequence designed for mathematics, science, and engineering students. Of the three courses examined, the graphics calculator course was most similar to the "traditional" calculus. Both covered essentially the same calculus topics. Both were taught in a lecture-recitation format (three 48-minute lectures and two 48-minute recitations each week) with new material presented by the instructor during lecture and problems from homework assignments and examinations discussed by a teaching assistant during recitation. Both required students to take three midterm examinations and a comprehensive final examination during the quarter. What differentiated the graphics calculator course from the traditional course was the amount of emphasis placed on the use of graphical representations. In the traditional course, little time was spent working with anything other than symbolic representations. In the graphics calculator course, course content, instruction, and assignments stressed using symbolic representations and graphical representations generated via graphics calculators. Symbolic and graphical representation were used on a regular basis when new material was presented and when problems were solved. Students were required to have a graphics calculator and use it during class, on homework assignments, and on portions of their examinations. In addition, concepts were presented in the course textbook both algebraically and graphically. The course textbook, *Calculus: A Graphing Approach, Volume 1* (Finney, Thomas, Demana, & Waits, 1993) included passages in most lessons where graphical representations of concepts were presented and discussed and exercises that required students to verifying or solving problems graphically. These exercises were designed to provide students with practice using three different approaches to solving problems that were introduced in the course.

1. Solve analytically, and <u>support</u> the results graphically.
2. Solve graphically, and <u>confirm</u> the results analytically.
3. Solve using a combination of graphical and analytic techniques.

(F. Demana, personal communication, April 18, 1994)

According to Demana, the first two approaches listed above were introduced to help students understand the role of analytical solution techniques in calculus and to recognize that a graph by itself does not constitute a proof. It should be noted that the university required 75% of the problems on each examination in this course to be equivalent to those used in the traditional course. This meant that the problems were to be solved using only symbolic methods (F. Demana, personal communication, April 18, 1994).

## Methodology

Data collected included class observations of each course, a posttest, and 36 interviews. Class observations were made by the researcher twice weekly, once during lecture and once during recitation. The purpose of the observations was to document use of numerical, graphical, and symbolic representations by instructors and students during class and on assignments and examinations. The posttest was used to assess students' abilities to use different representations when solving calculus problems. The problems on the posttest were designed by the researcher so that they might be solvable by any calculus students, no matter which of the three calculus course offered by the university they completed. These problems (see Appendix A) were pilot tested twice prior to the beginning of the study. Content validity for these problems was established by a panel of mathematicians and mathematics educators from across the United States.

The posttest was given during the final week of classes to 100 students from the three calculus courses, including 24 from the graphics calculator course. Because of time limitations - students had 30 minutes to complete the posttest - each student was asked to solve only two of the problems on the posttest. Problem 1 was assigned to all the students. The second problem was assigned randomly so approximately the same number of students from each course attempted problems 2, 3, and 4. Student were asked to solve these problems using whatever means they normally used to solve their homework or examination problems.

From the group of 100 students, 12 volunteers from each course were chosen to participate in one-on-one interview with the researcher. The interviews took place 4 to 8 weeks after the completion of the course. Each interview lasted from 25 to 45 minutes and was audiotaped. Each

student was paid $15 for participating in this portion of the study since the interviews were conducted during students' free time. The interviews were used to clarify how and determine why students used different representations when solving problems on the posttest. They also provided an opportunity to have students solve the problems using representations different from the ones they used during the posttest, and explain their reasoning for using these representations, while being observed and prompted by the researcher.

## Results

On the posttest and during the interviews, the graphics calculator students showed proficiency using graphical representations to solve problems but had trouble using symbolic representations and recognizing and making connections between graphical and symbolic representations. For example, nine of the 12 students interviewed were unable to describe how to use the first derivative to determine the local extrema of the function without some prompting, and only six students remember, without prompting, that the first derivative could be used to locate extrema. Some students knew they needed to take a derivative but were not sure how to use it to solve Problem 1a, as the following excerpts suggest.

Student G9: Probably set it equal to 0 [to solve Problem 1a algebraically].

*Interviewer:* *Set the function equal to 0?*

Student G9: Um hm. That's what I would have tried first, but I don't know if that would work or not. Maybe take a derivative, the derivative of it.

*Interviewer:* *Okay. What would you do with the derivative?*

Student G9: Hmm. That would - sheesh. I don't know.

*Interviewer:* *What information does the derivative give me?*

Student G9: I don't know.

Subject G1: I'd probably take, find the first derivative.

*Interviewer:* *Okay. So what would you do with the derivative? What information does it give me?*

Subject G1: I know how to find derivatives but I'm not sure exactly what they mean.

One student who had difficulties with Problem 1a did not remember ever being asked during the graphics calculator course to determine the maxima and minima of the graph of a function using analytic methods.

Subject G5: Taking the first derivative of the P(t) would give - uh. It should - it should give me the minimum and the maximum.

*Interviewer: Okay. Well, what would you do with the first derivative.... Do you remember any techniques or something you used to -*

Subject G5: Well, we never - uh - solved for a max - minimum and maximum with - uh - trigonometric functions in precalculus or high school, but we would - normally would have solved for x. But in this case, it would be t.

*Interviewer: This is the [derivative] you get. What equation would you solve?*

Subject G5: Yeah, I know. The equation'd be y equals this for the graph.

*Interviewer: Do you recall discussing ... some equation you created with the first derivative so that you could determine if something was a max or a min?*

Subject G5: Not really off the top of my head. I don't remember. I was -

*Interviewer: Okay. How about something like - do you remember doing this? Taking the first derivative and setting it equal to zero. And then solving for t.*

Subject G5: And solve for t? Uh - I don't remember from last quarter, but we did that. That's what we did in precalc in high school. I remember doing that.

*Interviewer: Okay.... So in your calculus class, you don't remember solving maximum and minimum problems other than looking at them graphically.*

Subject G5: No.... Not from last quarter.

Other students who knew to take the derivative and set it equal to zero could not explain why this procedure should be done to solve the problem.

*Interviewer: You mentioned confirm analytically. How would, what would you do to solve it analytically, or confirm it?*

Subject G6: Well, I had a problem with that last quarter. I had a problem with this section. Um - For the maximum, I'd try - I'd probably find the first derivative.... Solve it. I'd find the first derivative probably and solve it, make it equal 0.

*Interviewer: Why in particular did you choose setting the first derivative equal to zero in order to find the maximums and minimums? I mean, why - why zero?*

Subject G6: Why zero? That's the way I was taught - told to do it.

Subject G12: Find the derivatives. Yeah.... I would probably do that. I would probably find the derivative.

*Interviewer:* *Okay. And then do what?*

Subject G12: And then just find ... that equal to zero. Which would give you your max.

*Interviewer:* *So you set the derivative equal to zero?*

Subject G12: I think so.

*Interviewer:* *Why zero?*

Subject G12: Cause that where it would cross the *x*-axis.

*Interviewer:* *Why am I interested in that?... Why does it tell me a maximum occurs?*

Subject G12: Uh - cause that's just the way derivatives work.

*Interviewer:* *What information is the derivative giving me, so that when I find this maximum ... the derivative's zero?... Is there any way that maybe the derivative is somehow connect with slope? Does that ring a bell?*

Subject G12: No.

In the last excerpt, Subject G12's answers suggest a lack of understanding of the connection between the first derivative of a function at a point and its slope, or rate of change, at that point. This appeared to be a common difficulty amongst the graphics calculator students. For example, some students, like Subject G8, did not appear to recognize that how fast the population changed on one day (rate of change) corresponded to the value of the first derivative for that day.

*Interviewer:* *What am I asking for when I ask how fast the population's changing?*

Subject G8: How fast the numbers are changing. Your output.

*Interviewer:* *Does the derivative give me that information?*

Subject G8: I don't think so.

*Interviewer:* *For that matter, what does the derivative give me? When I talk about the derivative of this function, ... what information does it give me?*

Subject G8: Gives you the max and min.

*Interviewer:* *Does it give me anything else. For example, say I put .25 in [P'(t)]. I'll end up getting about 1000π. What does that number represent?*

Subject G8: The number of deer.

*Interviewer:* *If I put .25 in [the original function], that would give me the - uh - number of deer, but I'm putting it in the first derivative. What information does that give me?... [Doesn't it] tell me what the slope is?*

Subject G8: Slope. Yeah. The first derivative shows the slope of that.

*Interviewer:* *...Is there any way I can use the first derivative here?*

Subject G8: Yeah. See where it's zero. Wait a minute. Yeah. See where it's zero.

*Interviewer:* *Does that tell me how fast the population's changing on July 1st?*

Subject G8: I'm not sure. Just take - okay. Wait a minute. July 1st. That's halfway through the year? Then you just put - uh - .5 in the first derivative. that'll give you the slope there.

*Interviewer:* *Okay. Does that tell me how fast it's changing then? If I know the slope.*

Subject G8: You can make an estimate, but -let's see. Okay. I have the slope for it. Then could you take the slope and - uh - multiply it by -

*Interviewer:* *So once you know the slope, you want to multiply it by something. Would that give me how fast it's changing?*

Subject G8: [no response given]

Subject G8 seemed to have some understanding of the relationship between the value of the first derivative and the slope of the function at a particular point but apparently did not view the slope as representing the rate of change of the function at that point. Possible explanations for these and other difficulties exhibited by the graphics calculator students can be devised by viewing the effect on students of the instructional approach used in the graphics calculator course through the lens of the study's theoretical framework.

## Analysis of Findings

In the graphics calculator course, instruction emphasized use of graphical and symbolic representations to present concepts and solve problems. Such an approach could help students form more well-connected internal networks of knowledge containing graphical and symbolic representations for these different calculus concepts, rather than weakly-connected ones, since the approach provided them with opportunities to develop different mental representations of the concepts. However, class observations indicated that students solved few problems designed to

help them make connections between different representations of specific concepts. For the most part, such connections were pointed out to the students by the instructor when presenting material or solving problems during lecture and occasionally by the teaching assistant during recitation. This meant the students had little opportunity for the type of reflective abstraction during problem solving necessary for the construction of knowledge relating these representations. Thus, it is possible, if not likely, that some students formed weakly-connected internal networks of knowledge containing graphical and symbolic representations of different calculus concepts rather than more well-connected networks. This could explain why many graphics calculator students had difficulties working with symbolic representations and recognizing connections between graphical and symbolic representations; their internal networks of knowledge for the concepts addressed in this study were not well-connected or developed enough to handle these problems using more than just graphical representations.

It should be noted that during the interviews, comments made by the graphics calculator students suggested to the interviewer that many of them perceived the focus of the course to be on learning about and using only graphical representations of calculus concepts, and not symbolic representations. This perception may have influenced the development of the students' internal networks of knowledge related to symbolic representations and provides an alternate explanation as to why these students had difficulties using symbolic representations and recognizing connections between graphical and symbolic representations.

## Discussion

Findings from this study point out the need for further research on the effects of instruction emphasizing use of multiple representation in the presentation of concepts. Difficulties experienced by graphics calculator students suggest more than just viewing multiple representations of a concept is needed when developing the students' understanding of the concept. Having students solve problems designed to help them make connections between different representations of a concept, as observations indicated was done in the Calculus & *Mathematica* course, rather than having the connections pointed out to students by an instructor, as observation indicated was done

in the graphics calculator course, appears to be a key component in the development of students' understanding of these concepts . The need for further investigation becomes even more apparent when one considers technology that can create multiple, dynamically linked representations of different concepts *for* students will soon be readily available for use in the classroom.

The findings of this research also provides evidence that students "behave" as they are taught. The main emphasis of instruction in the graphics calculator course, as far students in this course were concerned, was on using graphical representations. During the posttest and interviews, these students used graphical representations and basically ignored symbolic representations. One student interviewed did not remember having been taught during the course how to solve problems where local extrema were determined symbolically. Four others stated that they thought they had solved such problems but could not remember for sure. This finding suggests the need for having instructors and students use a variety of representations when solving problems rather than favoring one particular representation which could help students develop more well-connected internal networks of knowledge associated with the concepts imbedded in the problems.

Finally, results from this study provide further evidence that the learning of calculus is not necessarily improved by simply adding technology such as a graphics calculator to the existing curriculum. The difficulties experienced by the graphics calculator students point out the importance of having students solve well-chosen problems designed to help them make connections between different representations of concepts that are provided by the technology. This suggests that as we consider revising the calculus curriculum, including calculus textbooks, we must be sure the revisions are done so that multiple representations and technology are *not* simply tacked onto the existing topics and problems, but are woven into a set of new topics and problems that emphasize multiple representations, connections between representations, and appropriate uses of the technology.

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## APPENDIX A - POSTTEST INSTRUMENT

1. The population of a herd of deer is given by the function

$$P(t) = 4000 - 500(\cos 2\pi t)$$

where $t$ is measured in years and $t = 0$ corresponds to January 1.

   a. When in the year is the population at its maximum? What is that maximum?

   b. When in the year is the population at its minimum? What is that minimum?

   c. When in the year is the population increasing the fastest?
   When in the year is the population decreasing the fastest?

   d. Approximately how fast the population is changing on the first of July?

2. Suppose N, the <u>total</u> number of people who have contracted a disease t days after its outbreak, is given by the formula $N = \dfrac{1{,}000{,}000}{1 + 5000e^{-0.1t}}$.

   a. In the long run, how many people will contract the disease?

   b. Is there a maximum number of people who will eventually contract the disease? Explain.

   c. Is there any day on which more than a million people fall sick? Half a million? Quarter of a million? (Note: You do not need to determine on what days these things happen.)

3. The table below gives U.S. population figures between 1790 and 1980.

| Year | Population (in millions) | Year | Population (in millions) | Year | Population (in millions) | Year | Population (in millions) |
|---|---|---|---|---|---|---|---|
| 1790 | 3.9 | 1840 | 17.1 | 1890 | 62.9 | 1940 | 131.7 |
| 1800 | 5.3 | 1850 | 23.1 | 1900 | 76.0 | 1950 | 150.7 |
| 1810 | 7.2 | 1860 | 31.4 | 1910 | 92.0 | 1960 | 179.0 |
| 1820 | 9.6 | 1870 | 38.6 | 1920 | 105.7 | 1970 | 205.0 |
| 1830 | 12.9 | 1880 | 50.2 | 1930 | 122.8 | 1980 | 226.5 |

   a. Approximately how fast was the population changing in the years 1900, 1945, and 1980?

   b. During what year(s) does it appear that the population growth was the greatest? Explain.

   c. Based on this data, what population would you predict for the 1990 census?

4. a. Show that $x > 2 \ln x$ for all $x > 0$.
   (Note: This is equivalent to showing that $e^x > x^2$ for all $x > 0$.)

   b. Is it true that $x > 3 \ln x$ for all $x > 0$?
   If not, estimate M such that $x > 3 \ln x$ for all $x > M$.

   c. What would you predict is the largest value of a for which $x > a \ln x$ for all $x > 0$? (Note: This is equivalent to predicting the largest value of a for which $e^x > x^a$ for all $x > 0$.)